

# Small $x$ Gluons in Nuclei and Hadrons

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## Abstract

We numerically study the effects of high gluon density at small  $x$  on the evolution of gluon distribution function in both hadrons and nuclei. Using a newly derived, Wilson renormalization group-based evolution equation which includes  $n$  to 1 gluon ladder fusion, we find significant reduction in nuclear gluon distribution function for large nuclei at zero impact parameter at energies relevant for RHIC and LHC experiments.

# 1 Introduction

Gluons are the most abundant partons in hadrons and nuclei at small  $x$  (high energy) and as such, will determine the behavior of many physical observables such as hadronic/nuclear cross sections through their initial distribution. Once the initial distribution of gluons in a hadron or nucleus is known, its change with  $x$  and  $Q^2$  can be predicted using the powerful machinery of perturbative QCD through the standard QCD evolution equations such as DGLAP [1] and BFKL [2]. Both DGLAP and BFKL evolution equations can describe the available experimental data quite well in a fairly broad range of  $x$  and  $Q^2$  with appropriate parameterizations. It is interesting to note that both equations predict a sharp growth of the gluon distribution function as  $x$  grows smaller which is clearly seen in the Deep Inelastic Scattering (DIS) experiments at HERA (see [3] for a recent review).

This sharp growth of the gluon distribution function will have to eventually slow down in order to not violate unitarity (Froissart [4]) bound on physical cross sections. Gluon recombination is believed to provide the mechanism which is responsible for this slow down or a possible saturation of the gluon distribution function at small  $x$ . In other words, the number of gluons at small  $x$  will be so large that they will spatially overlap and therefore, gluon recombination will be as important as gluon splitting and the standard evolution equations like DGLAP will have to be modified in order to take this into effect. A first step in this direction was taken in [5] by Gribov, Levin and Ryskin (GLR) who suggested the form of the first non-linear correction to DGLAP and identified the diagrams which contribute. In [6], Mueller and Qiu (MQ) made a Gaussian like ansatz for the gluon 4-point function which was taken to be proportional to the square of the gluon distribution function (2-point function). They then proceeded to calculate the numerical coefficient of the non-linear term.

It was shown in [6] that assuming reasonable values for  $R$ , a phenomenological parameter which could be taken to be either the proton or valence quark radius, gluon recombination effects were negligible in hadrons for not too small values of  $x$  but could be significant for large nuclei. It is perhaps helpful to mention that GLR/MQ and gluon

recombination based approaches in general are formulated in the infinite momentum frame. There has been much work inspired by the approach of GLR/MQ which show that gluon recombination leads to saturation of gluon density at small  $x$  [7]. However, GLR/MQ approach includes only the first non-linear term in the evolution equation and will not be valid at very small  $x$  where contribution of higher order terms will be as important as the first order correction and one will need to include them as well. Also, in the original approach of GLR/MQ there is no information about the impact parameter dependent gluon distributions and one typically assumes a factorization of the impact parameter and the usual gluon distribution function at all  $x$ .

In [8, 9], a new evolution equation for gluon distribution function (or any gluonic  $n$ -point function) was derived which is valid in the small  $x$  region. It was shown in [10] that the new equation reduces to all the standard evolution equations like BFKL, DLA DGLAP and GLR/MQ in the low density limit. In [11] double leading log limit of the new equation was considered and a closed form was obtained which generalizes the GLR/MQ equation. It was shown that this new equation slows down the growth of gluon distribution function at small  $x$  consistent with unitarity limits. In this paper, we will numerically solve this equation and investigate high gluon density effects on the evolution of gluon distribution. There are many interesting situations where understanding these effects should be useful.

Mini-jet production at high energy is an example where high gluon densities will play an important role. Mini-jets will be important at RHIC and will dominate at LHC over soft phenomena. Nuclear shadowing of initial gluon distribution and high gluon density could significantly reduce the initial mini-jet and total transverse energy production. Such reduced initial energy density will also affect the subsequent parton thermalization.

Another example is heavy quark production where high gluon density effects may make a dramatic difference specially at LHC. Since the probability for making a heavy quark pair is proportional to square of gluon density, any depletion in number of gluons will make a significant difference in the number of heavy quark pairs produced.

In section 2, we discuss the relation between gluon distributions in nuclei and hadrons followed by a brief description of Wilson renormalization group and effective action approach to high density/small  $x$  QCD. In section 4, We outline the semi-classical approach

for solving the general equation and use numerical methods to solve them. We finish by a discussion of our results and their experimental implications as well as the limitations of our approach.

## 2 Gluons in Hadrons and Nuclei

Gluon distribution function in a hadron,  $xG(x, Q^2)$ , has been studied quite extensively. Theoretically, once the initial distribution function at a given scale  $x_0$  and  $Q_0^2$  is known, one can calculate the distribution function at a different scale  $x$  and  $Q^2$  using the standard perturbative QCD-based evolution equations, for instance, the DGLAP equation. However, the initial distribution is non-perturbative and has to be supplied as an input to the evolution equation and is usually taken from parameterized experimental data. Alternatively, one can use the BFKL equation to study evolution of gluon distribution function with  $x$ . It is interesting to notice that both of these evolution equations predict a sharp rise of the gluon distribution function which would eventually lead to violation of unitarity (Froissart bound [4]).

Gluon distribution function of a proton can be measured indirectly in DIS experiments at HERA and elsewhere by measuring virtual photon-proton cross section  $\sigma_{\gamma^*p}$  where

$$\sigma_{\gamma^*p} \sim \frac{\alpha_s}{Q^2} xG(x, Q^2). \quad (1)$$

It should be kept in mind that eq. (1) is a leading twist relation and will break down when we consider higher twist terms in the evolution of the gluon distribution function. The theoretically predicted sharp growth of the gluon distribution function with  $x$  is observed experimentally at all  $Q^2$  as well as at fixed (and small)  $x$  with increasing  $Q^2$ . Even though DGLAP evolution equation fits the data very well, one can also use BFKL to explain the data and as of now, one can not experimentally distinguish between the two scenarios.

This sharp growth of gluon distribution function is expected to slow down eventually due to mutual interactions between gluons when they start to spatially overlap. This is usually referred to as saturation of gluon density and will happen when probability of two

gluons recombining into one is as large as the probability for a gluon to split into two gluons. In other words, when

$$\frac{\alpha_s}{Q^2} \frac{xG}{\pi R^2} \sim 1 \quad (2)$$

one has to include recombination effects which are neglected in DGLAP and BFKL evolution equations.

To illustrate this, one may write a generic evolution equation for gluon distribution function in the high density region as

$$\frac{\partial^2}{\partial y \partial \xi} xG \sim \left( \sum_{n=0}^{\infty} \left[ -\frac{\alpha_s}{Q^2} \frac{xG}{\pi R^2} \right]^n \right) xG \quad (3)$$

where  $y \equiv \ln 1/x$  and  $\xi \equiv \ln Q^2$ . The first term of the sum on the right hand side of eq.(3) is just the DGLAP equation whereas the second term is referred to as GLR/MQ since it was first investigated in [5] and its numerical coefficient was calculated by Mueller and Qiu [6]. An exact and formal evolution equation for gluon distribution function to all orders in gluon density in the high density (small  $x$ ) region was first derived in [8] in the infinite momentum frame. Whereas BFKL equation can be thought of as a ladder of reggized gluons, the general equation derived in [8] can be thought of as having  $n$  ladders of reggized gluon fusing into one and in this sense, it is the generalization of BFKL equation appropriate for the high gluon density region.

Until recently, there was no evidence that high density effects in a hadron were experimentally relevant even at the smallest  $x$  (highest energies) achieved at HERA in agreement with estimates made in MQ if one assumed reasonable values for the hadron/quark radius  $R$ . It should be emphasized that  $R$  is a purely phenomenological parameter and can not be derived from perturbative QCD. There is a very recent report on slope of the proton structure function  $F_2^p$  from HERA [12] which may be an indication of importance of higher twist effects in proton [13].

Gluon distribution function in a nucleus is intimately related to that in a hadron. Typically, one assumes that nucleus is a weakly bound system of nucleons so that one can neglect inter nucleon forces which is equivalent to taking the nucleus to be a dilute system

of nucleons in the transverse plane. This is a good approximation for hard processes in high energy nuclear collisions under normal conditions. Furthermore, if one assumes that density of gluons in a hadron is low, then one can simply relate distribution of gluons in nuclei and hadrons by

$$xG^A(x, Q^2) = A xG(x, Q^2) \quad (4)$$

where  $A$  is the atomic mass number,  $xG^A(x, Q^2)$  and  $xG(x, Q^2)$  are the nucleus and hadron (proton) gluon distribution functions.

Even if high density effects are not well established in hadrons, they are expected to be much more important for heavy nuclei. Non-linear terms in the evolution equation for gluon distribution function in a nucleus become appreciable at a larger  $x$  (lower energy) than for hadrons. In this sense, nucleus can be thought of as an amplifier of non-linear effects in QCD. These high density effects may very well be present in experiments planned at RHIC and LHC which underscores the crucial importance of a theoretically well-defined approach to nuclear gluon distribution function. Also, having nuclear beams at HERA would be of great help pinning down these effects and would be complementary to experiments planned for RHIC and LHC.

One of the advantages of our approach is that it can be used to investigate the gluon distribution function in both hadrons and nuclei and its impact parameter dependence without any assumptions on the form of impact parameter at small  $x$ . This will allow a systematic and rigorous determination of the change in the impact parameter of nuclear gluon distribution function with energy. To our knowledge, this is the first time that  $x$  and  $Q^2$  dependence of nuclear gluon distribution function as well as its impact parameter dependence have been derived from QCD in the high density region.

### 3 The General Evolution Equation

In this section we will briefly review the Wilson renormalization group and effective action approach to small  $x$  QCD as developed in [8]-[11], [14]-[16]. To make this paper self-contained, we will include some of the results reported above as needed. A few years ago,

McLerran and Venugopalan [14] suggested that for very large nuclei and/or at very small  $x$  one can use weak coupling, semi-classical methods to calculate structure functions. They considered a large nucleus in the infinite momentum frame and argued that as long as number of valence quarks per unit area per unit rapidity is large, they can be treated as static, classical sources of color charge to which the long wavelength gluonic fluctuations (small  $x$  gluons) couple. In order to perform color averaging over the hadron/nucleus state, they assumed a Gaussian weight for color configurations. They proceeded to solve Yang-Mills equations of motion and calculated the gluon distribution function in lowest order in the coupling constant. Quantum corrections to the classical result were computed in [15] and analogous to standard perturbation theory, large logarithmic factors ( $\ln 1/x$ ) were encountered which necessitated a formalism which would resum these large logarithmic factors in presence of a non-trivial background (classical) field.

In [10], McLerran-Venugopalan action was generalized as the following

$$S = -\frac{1}{4} \int d^4x G_a^{\mu\nu} G_{\mu\nu}^a + i \int d^2x_\perp F[\rho^a(x_\perp)] \\ + \frac{i}{N_c} \int d^2x_\perp dx^- \delta(x^-) \rho^a(x_\perp) \text{tr} T_a W_{-\infty, \infty}[A^-](x^-, x_\perp)$$

where  $W$  is the Wilson line in the adjoint representation along the  $x^+$  axis

$$W_{-\infty, \infty}[A^-](x^-, x_t) = P \exp \left[ -ig \int dx^+ A_a^-(x^+, x^-, x_t) T_a \right]. \quad (5)$$

The nucleus/hadron is represented by an ensemble of color charges localized in the plane  $x^- = 0$  with the (integrated across  $x^-$ ) color charge density  $\rho(x_\perp)$ . Statistical weight of a configuration  $\rho(x_\perp)$  is given by

$$Z = \exp\{-F[\rho]\} \quad (6)$$

In light cone gauge  $A^+ = 0$  and at the tree level, the chromoelectric field is determined by the color charge density through the equations

$$G^{+i} = \frac{1}{g} \delta(x^-) \alpha_i(x_\perp) \quad (7)$$

where the two dimensional vector potential  $\alpha_i(x_\perp)$  is "pure gauge" and is related to the color charge density by

$$\begin{aligned}\partial_i \alpha_j^a - \partial_j \alpha_i^a - f^{abc} \alpha_i^b \alpha_j^c &= 0 \\ \partial_i \alpha_i^a &= -\rho^a\end{aligned}\tag{8}$$

One can then consider quantum fluctuations in background of this classical field and separate hard and soft modes (in light cone longitudinal momenta) of the fluctuations, keeping terms quadratic in hard fluctuations. Integrating out the hard modes generates the renormalization group equation which has the form of the evolution equation for the statistical weight  $Z$  [8]

$$\frac{d}{dy} Z = \alpha_s \left\{ \frac{1}{2} \frac{\delta^2}{\delta \rho(u) \delta \rho(v)} [Z \chi(u, v)] - \frac{\delta}{\delta \rho(u)} [Z \sigma(u)] \right\}\tag{9}$$

In the notation used in Eq. (9), both  $u$  and  $v$  stand for pairs of color index and transverse coordinate with summation and integration over repeated occurrences implied. The evolution in this equation is with respect to the rapidity  $y$ , related to the Bjorken  $x$  by  $y = \ln 1/x$ .

The quantities  $\chi[\rho]$  and  $\sigma[\rho]$  have the meaning of the mean fluctuation and the average value of the extra charge density induced by the hard modes of quantum fluctuations. They are functionals of the external charge density  $\rho$ . The explicit expressions have been given in [8].

Using this equation for the statistical weight  $Z$ , one can derive evolution equations for  $n$ -point functions of gluon field [8]. In [11], double leading log limit of the 2-point function (gluon distribution function) was investigated and shown to be

$$\frac{d}{dy} \langle \alpha_i^a(X) \alpha_i^a(Y) \rangle = 4\alpha_s \left[ \langle X | \frac{\alpha^2}{\partial_\perp^2 + 2\alpha^2} | Y \rangle \right]^{aa}\tag{10}$$

In the high density limit where  $\alpha^2 \gg \partial_\perp^2$ , one can neglect the derivative term in the denominator above and the right hand side is a constant which leads to the gluon distribution function growing only logarithmically with  $x$  (energy) consistent with unitarity. In the low density limit where  $\alpha^2 \ll \partial_\perp^2$ , one can expand the denominator in the above



equation. The first term of the expansion gives the DGLAP equation. Furthermore, if one assumes a factorization of the 4-point function in terms of the 2-point function (as assumed by GLR/MQ), one recovers the GLR/MQ equation [6]. This is equivalent to ignoring all correlations between gluon fields except that they are constrained to be in an area of  $\pi R^2$ . With these assumptions, one can actually perform the color averaging in (10) which leads to

$$\frac{\partial^2}{\partial y \partial \xi} xG(x, Q, b_\perp) = \frac{N_c(N_c - 1)}{2} Q^2 \left[ 1 - \frac{1}{\kappa} \exp\left(\frac{1}{\kappa}\right) E_1\left(\frac{1}{\kappa}\right) \right] \quad (11)$$

where

$$\kappa = \frac{2\alpha_s}{\pi(N_c - 1)Q^2} xg(x, Q, b_\perp) \quad (12)$$

and  $E_1(x)$  is the exponential integral function defined as [17]

$$E_1(x) = \int_0^\infty dt \frac{e^{-(1+t)x}}{1+t}, \quad x > 0 \quad (13)$$

In the low density limit, one can expand equation (11). Keeping the first term, we get

$$\frac{\partial^2}{\partial y \partial \xi} xG(x, Q, b_\perp) = \frac{N_c \alpha_s}{\pi} xG(x, Q, b_\perp) \quad (14)$$

which is the DLA DGLAP (small  $x$  limit of DGLAP) equation. In the high density limit, eq. (11) gives

$$\frac{\partial^2}{\partial y \partial \xi} xG(x, Q, b_\perp) = \frac{N_c(N_c - 1)}{2} Q^2 \quad (15)$$

which leads to a gluon distribution of the form

$$xG(x, Q, b_\perp) \sim Q^2 \ln 1/x. \quad (16)$$

Let's consider the impact parameter dependent gluon distribution function  $xG(x, Q, b_\perp)$  which is related to gluon distribution function  $xG(x, Q)$  by

$$xG(x, Q) = \int d^2 b_\perp xG(x, Q, b_\perp). \quad (17)$$

It is usual to factor out the impact parameter dependence of the distribution function and write  $xG(x, Q, b_\perp) = S(b_\perp) xG(x, Q)$  where  $S(b_\perp)$  is the nucleus/nucleon shape function and can be taken to be a Gaussian

$$S(b_\perp) = \frac{e^{-b_\perp^2/R^2}}{\pi R^2} \quad (18)$$

so that  $\int d^2b_\perp S(b_\perp) = 1$ . This factorization introduces the phenomenological parameter  $R$  which is taken to be the nuclear/hadronic radius. As long as one is using the DGLAP evolution equation, this parameter does not come into play since DGLAP equation is linear in gluon density. However, as we consider the first non-linear term in the evolution equation as in GLR/MQ equation, parameter  $R$  becomes relevant and one needs to define it precisely. Since in GLR/MQ impact parameter dependence is factorized, one can only make plausible estimates of  $R$ .

In general, this factorization of impact parameter will break down with evolution in  $x$  and  $Q^2$  simply because gluon densities are expected to be higher in the central ( $b_\perp = 0$ ) region than the peripheral ( $b_\perp \sim R$ ) region and so therefore will evolve differently. This would lead, in the general case, to a breakdown of factorization of impact parameter and Gaussian ansatz for the nucleus/nucleon shape function. In the present case where we are working in the double logarithmic region, the Gaussian ansatz for the shape function should still hold but the area (or radius  $R$ ) would change with  $x$  and  $Q^2$ . This basically amounts to the rise of perturbative cross sections with energy. In this work, we factorize the impact parameter only at the starting point of our evolution  $x_0$  and  $Q_0^2$  where non-linear effects are believed to be experimentally absent. The evolution equation will then predict the change of this "area" with energy.

## 4 Solving the General Equation

In this section we will outline the procedure to numerically solve the general equation (11). We will use the method of characteristics which converts a partial differential equation to a set of coupled ordinary differential equations [18] (see also [7, 19, 20] for an illustration of

this method). We will also use MQ [6] normalization of 4-point function in terms of 2-point functions in order to facilitate comparison of our results with those where one includes only the first non-linear term in the evolution equation. This amounts to a simple rescaling of our gluon distribution function

$$xG(x, Q, b_\perp) \rightarrow \frac{N_c(N_c - 1)\pi^3}{6} xG(x, Q, b_\perp). \quad (19)$$

In the following, we will closely follow the derivation of Ayala et al. in [7] and rewrite (11) in terms of the density factor  $\kappa$  so that one gets

$$\frac{\partial^2}{\partial y \partial \xi} \kappa + \frac{\partial}{\partial y} \kappa = \frac{N_c \alpha_s}{\pi} \left[ 1 - \frac{1}{\kappa} \text{Exp}\left(\frac{1}{\kappa}\right) E_1\left(\frac{1}{\kappa}\right) \right] \quad (20)$$

where the rescaled  $\kappa$  is now

$$\kappa = \frac{N_c \alpha_s}{\pi} \frac{\pi^3}{3Q^2} xG(x, Q, b_\perp). \quad (21)$$

In the semi-classical approximation, one can write the solution to (20) as

$$\kappa \equiv e^S \quad (22)$$

and neglect

$$\frac{\partial^2 S}{\partial y \partial \xi} \ll \frac{\partial S}{\partial y} \frac{\partial S}{\partial \xi}.$$

Defining  $\frac{\partial S}{\partial y} \equiv \omega$  and  $\frac{\partial S}{\partial \xi} \equiv \gamma$ , we get

$$\omega(\gamma + 1) = \Phi(S) \quad (23)$$

where

$$\Phi(S) \equiv \frac{N_c \alpha_s}{\pi} e^{-S} \left[ 1 - e^{-S+e^{-S}} E_1(e^{-S}) \right] \quad (24)$$

In terms of these variables, the set of characteristic equations become

$$\frac{dS}{dy} = \frac{2\gamma + 1}{(\gamma + 1)^2} \Phi \quad \frac{d\xi}{dy} = \frac{1}{(\gamma + 1)^2} \Phi \quad \frac{d\gamma}{dy} = \frac{\gamma}{\gamma + 1} \frac{\partial \Phi}{\partial S}. \quad (25)$$

Notice that these equations are identical in form to those in [7] except that our function  $\Phi(S)$  is different and will therefore result in different solutions.

In order to solve these equations, We need some initial conditions. Since they are first order ordinary differential equations, we will need to specify their initial values denoted by  $S_0$ ,  $\gamma_0$  and  $\xi_0$  at some initial point  $y_0$ . In order to clarify these initial conditions, it is helpful to write them explicitly in terms of the gluon distribution function

$$\begin{aligned} S_0 &= \ln \left[ \frac{N_c \alpha_s}{\pi} \frac{\pi^3}{2Q^2} xG(x_0, Q_0, b_\perp) \right] \\ \gamma_0 &= \frac{\partial}{\partial \xi} \ln xG(x, Q, b_\perp)|_{x_0, Q_0} - 1 \\ \xi_0 &= \ln Q_0^2 \end{aligned} \tag{26}$$

We will choose the initial  $x_0$  and  $Q_0$  such that the non-linear terms are negligible in the evolution equation. For a nucleon, this requirement is not very restrictive since non-linear effects are small in a broad range of  $x$  and  $Q^2$ . In a nucleus, however, it is known experimentally that there is a narrow range of  $x$  such that the shadowing ratio  $S = \frac{F_2^A}{F_2^N} \sim 1$  so that we will restrict our initial point  $x_0$  and to lie in this region. From experimental data [21, 22], it appears that  $x_0 \sim 0.05 - 0.07$  is a reasonable value so that for the sake of definiteness, we will choose  $x_0 = 0.06$  but have checked that our results are not very sensitive to variation of  $x_0$  in this range. We also choose  $Q_0 = 0.7$  (in practice, most characteristic lines start at a higher  $Q_0$ ) for the following reasons: quite surprisingly, all HERA data can be explained by starting at such low values of  $Q_0$  so that perturbative QCD seems to hold at such small values in DIS (GRV parameterization of parton densities start at  $Q_0 = 0.5 \text{ GeV}$ ). Also, since we are using the method of characteristics to solve these equations, it is useful to start from a low  $Q_0$  in order to be able to find the characteristic lines in a broad range of  $x$ . Most importantly, we want to get an upper limit on the amount of perturbative shadowing generated so that it is helpful to start from as low virtualities as allowed by perturbative QCD.

Having chosen our initial point  $x_0$  and  $Q_0$ , we use CTEQ parameterization of the proton gluon density to get  $x_0 G^N(x_0, Q_0)$ . Also, at the initial point  $x_0$  and  $Q_0$ , we assume

that factorization of the impact parameter is valid as discussed in some length earlier

$$x_0 G(x_0, Q_0, b_\perp) = S(b_\perp) x_0 G(x_0, Q_0).$$

Putting everything together, at the initial point  $x_0$  and  $Q_0$ , the impact parameter dependent gluon distribution function in nuclei and hadrons can be written as

$$x_0 G^A(x_0, Q_0, b_\perp) = A \frac{e^{-b_\perp^2/R_A^2}}{\pi R_A^2} x_0 G^N(x_0, Q_0) \quad (27)$$

and

$$x_0 G^N(x_0, Q_0, b_\perp) = \frac{e^{-b_\perp^2/R^2}}{\pi R^2} x_0 G^N(x_0, Q_0) \quad (28)$$

where  $R_A^2$  and  $R^2$  are the nuclear and hadronic areas at the initial point  $x_0$  and  $Q_0$ . This completely fixes our initial conditions for solving the set of coupled ordinary differential equations in (25).

We would like to emphasize that the factorization of the impact parameter dependent distribution into a Gaussian shape function  $S(b_\perp)$  and the standard gluon distribution function  $xG(x, Q)$  is done only at the initial point and in principle will not hold when one goes to small  $x$  where solution of the evolution equation will determine its functional form. Here, we will mostly work at zero impact parameter since that is where gluon densities and hence non-linearities are most important but we intend to investigate impact parameter dependence of our results in more detail in future work.

Having determined our initial conditions, we use the 4th order Runge-Kutta method to solve the set of characteristic equations in (25). Some of the characteristic lines are shown in Figure (1) for illustration. All the lines start at  $x_0 = 0.06$  and end at  $Q = 10 GeV$ . In order to find the gluon distribution function  $xG(x, Q, b_\perp)$  at a given  $x$ , one would need to vary the initial  $Q_0$  until the corresponding characteristic passing through the given  $x$  and  $Q$  is found. For the range of  $x$  and  $Q$  considered here, variation of  $Q_0$  is between  $0.8 - 1.5 GeV$  so that even though the evolution formally starts at a low value of  $Q_0 = 0.7 GeV$ , the actual initial  $Q_0$  is higher.

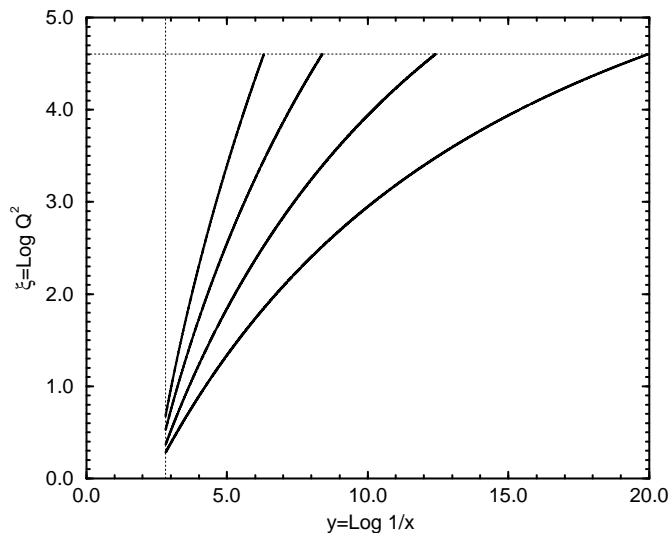


Figure 1: Some characteristic lines of equation (25) starting at  $x_0 = 0.06$  ( $y_0 = 2.81$ ).

In Figures (2) and (3), we show the ratio

$$R(x, Q, b_\perp) = \frac{xG^{JKLW}(x, Q, b_\perp)}{xG^{DGLAP}(x, Q, b_\perp)} \quad (29)$$

at  $b_\perp = 0$  for both  $A = 1$  and  $A = 200$  at  $Q = 2 \text{ GeV}$  and  $Q = 5 \text{ GeV}$ . We have also taken  $R_A = 5 \text{ fm}$ ,  $R = 1 \text{ fm}$  and  $\alpha_s = .25$ . Here,  $xG^{JKLW}$  refers to solution of equation (11) while  $xG^{DGLAP}$  is the solution of (14). For a proton, we get a 15 – 20% reduction in the number of gluons at  $x \sim 10^{-4}$  as compared to DLA DGLAP while for a Gold or Lead nucleus, there is a 50 – 55% reduction at  $x \sim 10^{-4}$ . It is expected that these results will have some dependence on the numerical values of the hadron or nucleus radius as well as the coupling constant  $\alpha_s$ . For example, one can find values like  $R = .5 \text{ fm}$  for radius of proton and so on in the literature. Also, one could use a more realistic shape function like Woods-Saxon rather than a Gaussian but these changes are not expected to make more than a few percent change in our results. In this work, we are interested in the overall features of the non-linear effects and will investigate these details later.

In Figure 4, we show the  $Q$  dependence of this ratio at different  $x$  for  $A = 200$ . To make comparison easier, we normalize  $R = 1$  at  $Q = 1$ . For a proton, the  $Q$  dependence was found to be negligible and is not shown.

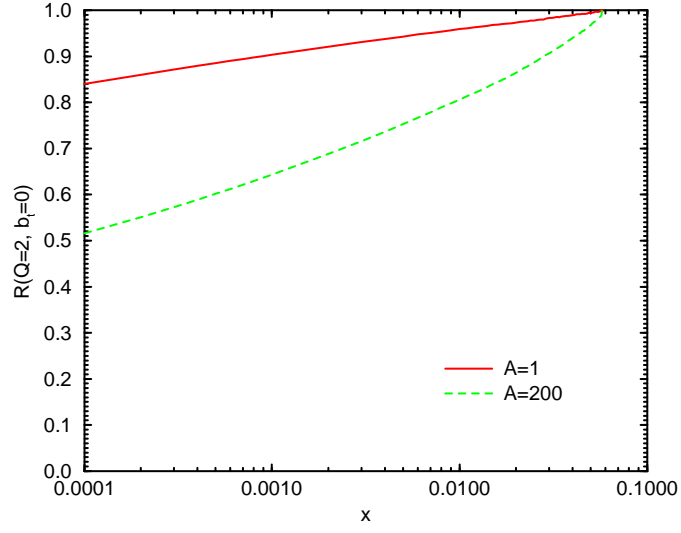


Figure 2:  $R(x, Q, b_{\perp})$ , as defined in (29) at  $Q = 2 \text{ GeV}$  and  $b_{\perp} = 0$ .

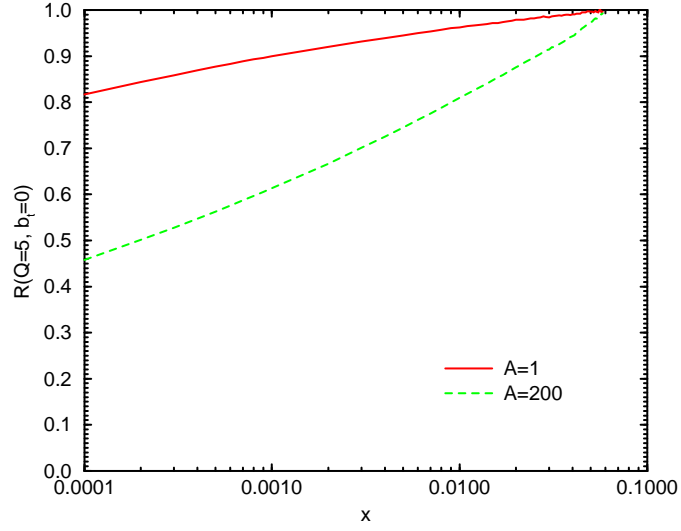


Figure 3: Same as in Figure (2) at  $Q = 5 \text{ GeV}$ .

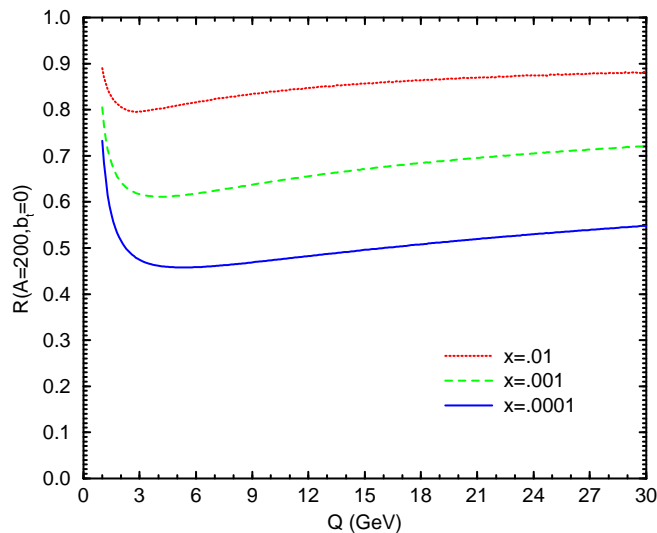


Figure 4:  $R(x, Q, b_{\perp})$  as a function of  $Q$  at different  $x$  for  $A = 200$ .

As is seen, the non-linearities become less important at higher  $Q$ . However, the low  $Q$  behavior is peculiar since one expects a monotonous decrease of  $R$  with decreasing  $Q$  while it is seen to eventually increase with decreasing  $Q$  and tend to 1. This dip in  $Q$  is a consequence of using the method of characteristics to solve partial differential equations and is not an artifact of our formalism (see, for example, Figs. 23, 24 in [7]).

To find the gluon distribution function  $xG(x, Q)$  at  $Q$ , one needs to find the characteristic line of the corresponding evolution equation passing through that value of  $Q$  by finding the value of  $Q_0$  from which the desired characteristic line starts.  $xG^{JKLW}$  and  $xG^{DGLAP}$  satisfy different evolution equations and therefore have different characteristic lines which start at different values of  $Q_0$  in order to reach the point  $Q$ . In other words, the initial starting  $Q_0$  is never exactly the same for the two evolution equations. This would not be important if the distribution functions at the initial  $Q_0$  were slowly varying which is not the case for  $xG^{JKLW}(x, Q)$  since it includes high density effects. To make a completely self-consistent treatment of perturbative shadowing possible, one would need to start from parameterizations of parton distributions which include initial non-perturbative shadowing such as [23] rather than CTEQ, GRV or MRS. We intend to do this when we



study perturbative gluon shadowing in nuclei in the near future.

In Figures (5) and (6), we show the  $A$  and  $b_\perp$  dependence of our results at fixed  $Q$  for different values of  $x$ . As is seen, non-linear effects set in rather quickly, specially at low  $x$ , after which they increase slowly.

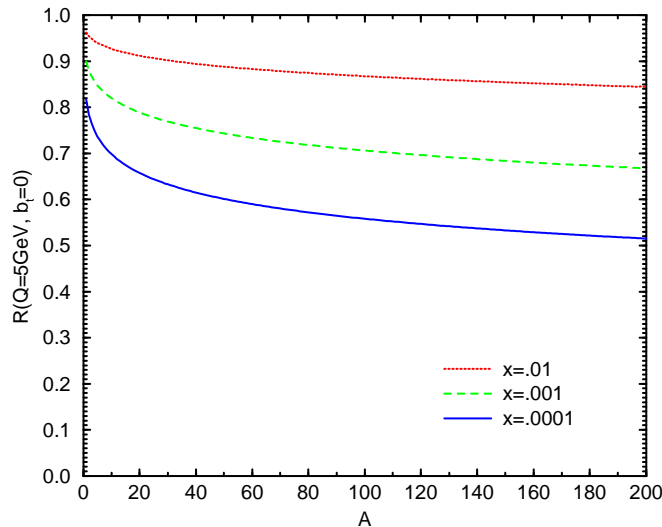


Figure 5:  $R(x, Q, b_\perp)$  as a function of  $A$  at  $Q = 5GeV$  for different  $x$ .

These figures clearly show the importance of the non-linear terms, specially for a large nucleus, at values of  $x$  which will be reached in the upcoming experiments at RHIC and LHC. These non-linear effects will be manifest in terms of shadowing of nuclear gluon distribution function and will have to be taken into account at future high energy heavy ion experiments. We are currently investigating shadowing of nuclear gluon distributions [24] using this formalism.

## 5 Discussion

We have investigated the  $x$  and  $Q$  dependence of gluon recombination effects on the evolution of impact parameter dependent gluon distribution function in hadrons and nuclei using a general evolution equation which takes  $n \rightarrow 1$  gluon ladder fusion into account. Using the

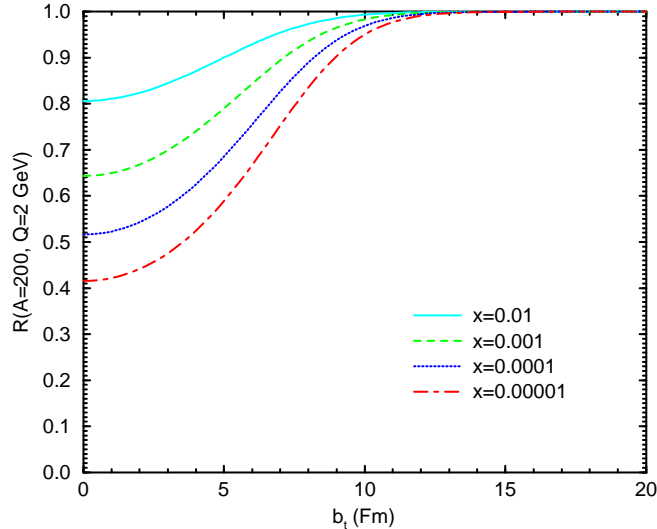


Figure 6:  $R(x, Q, b_{\perp})$  as a function of  $b_{\perp}$  at different values of  $x$ .

method of characteristics, we numerically solved the general evolution equation and found that gluon recombination effects are very important specially for central ultra-relativistic heavy ion collisions coming up at RHIC and LHC.

A detailed knowledge of nuclear gluon distribution will be essential in understanding the outcome of RHIC and LHC experiments. With our formalism and with the solution to the general evolution equation at hand, we can investigate several aspects of these distributions. The first thing to consider is shadowing of nuclear gluon distributions using the present formalism. In addition to the usual shadowing ratio defined as  $\frac{xG^A(x, Q)}{AxG(x, Q)}$ , we can investigate impact parameter dependence of this ratio. This will give information about shadowing of gluon distributions in the peripheral as well as the central region. Studying  $x$  and  $Q$  dependence of the average impact parameter will determine how the effective area (cross section) of a nucleus changes with increasing energy of nuclear collisions. The  $A$  dependence of shadowing ratio at small  $x$  can also be determined rigorously without any model dependent assumptions [6]. One should also integrate over the impact parameter eventually in order to make possible a comparison with other approaches which take only the first non-linear term into account. Work in this direction is in progress [24].

One could also consider the effect of shadowing of nuclear gluon distributions on physical cross sections like Drell-Yan [25] and heavy quark production. To do this, one would need to include the relevant higher twist effects not only in the evolution of gluon distribution (as is done here) but also in the relation between the cross section (or  $F_2$ ) and gluon distribution function. In [26], the all twist structure function,  $F_2$ , was computed in the infinite momentum frame (see also [7, 27] for a similar calculation in the lab frame) so that all one has to do is to merge the two results [28].

There are a few issues which need to be analyzed further. First, one should investigate our choice of the initial virtuality  $Q_0$  in solving the evolution equation. We took a low value of  $Q_0 = 0.7 \text{ GeV}$  (see the comments before eq. (29)) because it seems to be consistent with DIS data from HERA and in order to get maximum perturbative shadowing of gluon distribution possible. However, from Figure (4), we see that  $R$  decreases rather sharply at low values of  $Q$  before it starts to increase with increasing  $Q$  as it must as a high twist effect. This means that our treatment, strictly speaking, is not self-consistent at low values of  $Q$ . In other words, starting from high values of  $Q$  and decreasing  $Q$ , we should have a monotonous decrease of  $R$  as is seen in Figure (4). Further decrease of  $Q$  leads to  $R$  increasing in order to match our initial condition that  $R(x_0, Q_0) = 1$ . This indicates that one should include some initial non-perturbative shadowing so that  $R(x_0, Q_0) \neq 1$ . Unfortunately, this requires detailed knowledge of  $x$ ,  $Q$ ,  $A$  and  $b_\perp$  dependence of gluon distribution function at the initial point which necessitates use of many model dependent assumptions. There are a number of approaches including vector meson dominance, Pomeron exchange models, etc. ( see [29] and references therein) which one could adopt to address this issue. This is beyond the scope of present work and will be pursued in future.

We are also going to study the dependence of our results on our choice of CTEQ parameterization of gluon densities by repeating this calculation using other available parameterizations like MRS and GRV. However, since we used the parameterized gluon distributions at a fairly high initial  $x_0 \sim 0.06$  and because parameterization dependence of gluon distribution function becomes noticeable only at small values of  $x$ , we do not believe our results will be sensitive to choice of parameterization.

Another point to keep in mind is that we have been working in the leading log approximation and therefore have taken  $\alpha_s$  to be a constant. It is known that DLA DGLAP with fixed  $\alpha_s$  overestimates the gluon number density at small  $x$  and to get good agreement with experimental data from HERA, one needs to include next to leading order corrections to DGLAP. However, our general evolution equation is derived in the leading log approximation and will be modified if one goes beyond the leading log approximation. One such modification due to next to leading order corrections is to cause running of  $\alpha_s$ , but this may not be the only effect or even the dominant effect so that to be theoretically consistent, we have kept everything at the leading log approximation level. However, so long as we are working with ratios of distributions, we think next to leading order corrections to this ratio will not be large and a leading order calculation such as this should be adequate.

Our emphasis in this work has been on theoretical self-consistency so that we have not made any attempt to fit or reproduce experimental data. The main reason is that until now, there has been little effort made to calculate nuclear gluon distribution function directly from QCD without resorting to elaborate modeling. Also, there is not much experimental data available on nuclear gluon distribution function. What is experimentally measured is the structure function  $F_2$  and to get the gluon distribution function, one takes logarithmic derivative of  $F_2$

$$xG(x, Q) \sim \frac{d}{d \log Q^2} F_2. \quad (30)$$

As mentioned earlier, this is a leading twist relation and will not be valid in a general all twist calculation such as ours. By using eq. (30) to extract the gluon distribution function experimentally, one is implicitly assuming that higher twist effects are not present. To be theoretically consistent, one should figure out how to extract gluon densities from experimental data allowing for higher twist effects. Until this is done, in our opinion, one should develop a self-consistent approach derived from fundamental theory with well defined approximations and with as less model dependence as possible. This is the approach adopted in this work.

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